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Cylindrical Implosion in Shallow Water Theory

by

CATHLEEN S. MORAWETZ

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CYLINDRICAL IMPLOSION IN SHALLOW WATER THEORY

By Cathleen S. Morawetz

We seek here a theory to describe the behavior of a cylindrical shock in shallow water as it contracts to the center and then expands again.

The corresponding problem for gases has been worked out by Guderley [1] for strong shocks. Although shallow water theory corresponds to a gas with $\gamma = 2$, the cylindrical reflection theory breaks down because the shock conditions are essentially different.

The solution given here is to be valid only in the neighborhood of the center and for times close to the time when the shock hits the center. This is also true of the reflection solutions in gases. However, Guderley's solution is an exact solution of the differential equations corresponding to a shock of prescribed velocity, which becomes infinite at the center, impinging on a region of zero pressure. The solution here for a shallow water implosion is an exact solution for the shallow water approximation only if the water ahead of the shock is of zero depth. This is quite unrealistic. However, for arbitrary depth ahead the given solution satisfies the differential equations to zero order in $(\log t)^{-1}$ and the boundary conditions corresponding to a shock velocity which becomes infinite in a prescribed way, at the center. It is expected to represent the flow except on

the shock paths.

In Section 1, we shall find the differential equations and describe the method of finding special solutions. In Section 2, we shall determine the boundary conditions and find the solution corresponding to zero depth ahead of the shock. In Section 3, we shall modify this solution and show that the resultant solution satisfies the differential equations and the shock conditions near the center, up to terms of order $(\log t)^{-1}$. In Section 4, we discuss a computed flow.

1. Special Solutions

According to shallow water theory [2], the depth Z and the horizontal velocity u of water satisfy the equations,

$$
\begin{aligned}
&u_t + u\,u_r + gZ_r = 0 \\
&Z_t + u\,Z_r + Z\,u_r + r^{-1}uZ = 0
\end{aligned}
\tag{1}
$$

if the flow has cylindrical symmetry; r is the distance from the origin, t time, and g the acceleration of gravity.

Setting $gZ = c^2$, equations (1) become

$$
\begin{aligned}
&u_t + uu_r + 2cc_r = 0 \\
&2c_t + 2uc_r + cu_r + r^{-1}uc = 0 .
\end{aligned}
\tag{2}
$$

Note that c has the dimensions of velocity and is the speed of propagation of small disturbances.

There are various special solutions of (2) but only one class need be considered here. All others lead to physically impossible solutions. We reject for example solutions for which $Z \longrightarrow 0$ as $t \longrightarrow \infty$ or for which $Z \longrightarrow \infty$ or $u \longrightarrow \infty$ for $t = 0$, $r \neq 0$. We must permit solutions that are singular at $r = 0$, $t = 0$.

The solutions considered are the so-called progressive waves, [3, p.424]

$$u = \frac{r}{t} U(\eta) \tag{3}$$

$$c = \frac{r}{t} C(\eta)$$

where

$$\eta = tr^{-1} . \tag{4}$$

Then U and C satisfy

$$(1 - U)\eta U_\eta - 2C\eta C_\eta + U(U - 1) + 2C^2 = 0 \tag{5}$$

$$- \eta U_\eta + 2(1 - U)C^{-1}\eta C_\eta + 4U - 2 = 0$$

or

$$((1 - U)^2 - C^2)\eta U_\eta = A \tag{6}$$

$$((1 - U)^2 - C^2)\eta C_\eta = BC$$

where $A = U((1 - U)^2 - 2C^2)$, $B = - C^2 - (1 - U)(\frac{3}{2}U - 1)$ or eliminating η we get a nonlinear ordinary differential equation relating U and C :

$$\frac{dC}{dU} = \frac{BC}{A} . \tag{7}$$

Then a solution of (1) is given by (3) if $C(U)$ is a solution of (7) and η is found from (6):

$$\frac{d\eta}{\eta} = \frac{(1-U)^2 - C^2}{A}\, dU \,. \tag{8}$$

From (8) and (4) we see, as in the case of gases that no solution may cross $C^2 = (1 - U)^2$ unless $A = 0$, i.e. $U = 0$, $C = \pm 1$, or $U = 1$, $C = 0$. Otherwise $d\eta$ changes sign or for fixed r, dt changes sign which is impossible.

The field of the solutions of (7) is to be investigated particularly in the neighborhood of the singular points where both BC and A vanish. These points are

$$U = 1, \quad C = 0$$

$$U = \frac{1}{2}, \quad C = \pm \frac{1}{2\sqrt{2}}$$

$$U = 0, \quad C = \pm 1$$

$$U = 0, \quad C = 0 \,.$$

First from (3), to have a solution valid on the line $t = 0$ we must have $U = 0$, $C = 0$ for $\eta \longrightarrow 0$. Hence we need consider only solutions of (7) which pass through $(0, 0)$. From (7) we see that near $(0, 0)$

$$\frac{dC}{dU} \sim \frac{C}{U}$$

and hence $U^{-1}C \sim$ constant represent solutions. If the solutions are to be continuous across $t = 0$ then the slope

of the solution curve must be continuous through the origin.

Near $U = 1$, $C = 0$ we find

$$\frac{dC}{dU} = \frac{C(C^2 + \frac{1}{2}(1-U))}{2C^2 - (1-U)^2} .$$

By the theory of ordinary nonlinear differential equations, this singularity is a node. Except for the solution $C = 0$, all solutions enter $U = 1$, $C = 0$ on the lines $C = \pm \frac{1}{2}(1-U)$. To determine the direction of increasing time, $d\eta > 0$, we find on $C^2 = \frac{1}{4}(1 - U)^2$ that both $(1 - U)^2 - C^2$ and A are positive. Hence $d\eta > 0$ for $\eta dU > 0$.

The point $(\frac{1}{2}, \pm \frac{1}{2\sqrt{2}})$ is a saddle point singularity. The singularity at $(0, \pm 1)$ is again a node.

The field of solutions is sketched in Figure 1, cf. [3], p. 426, with the direction of increasing time indicated by the arrow.

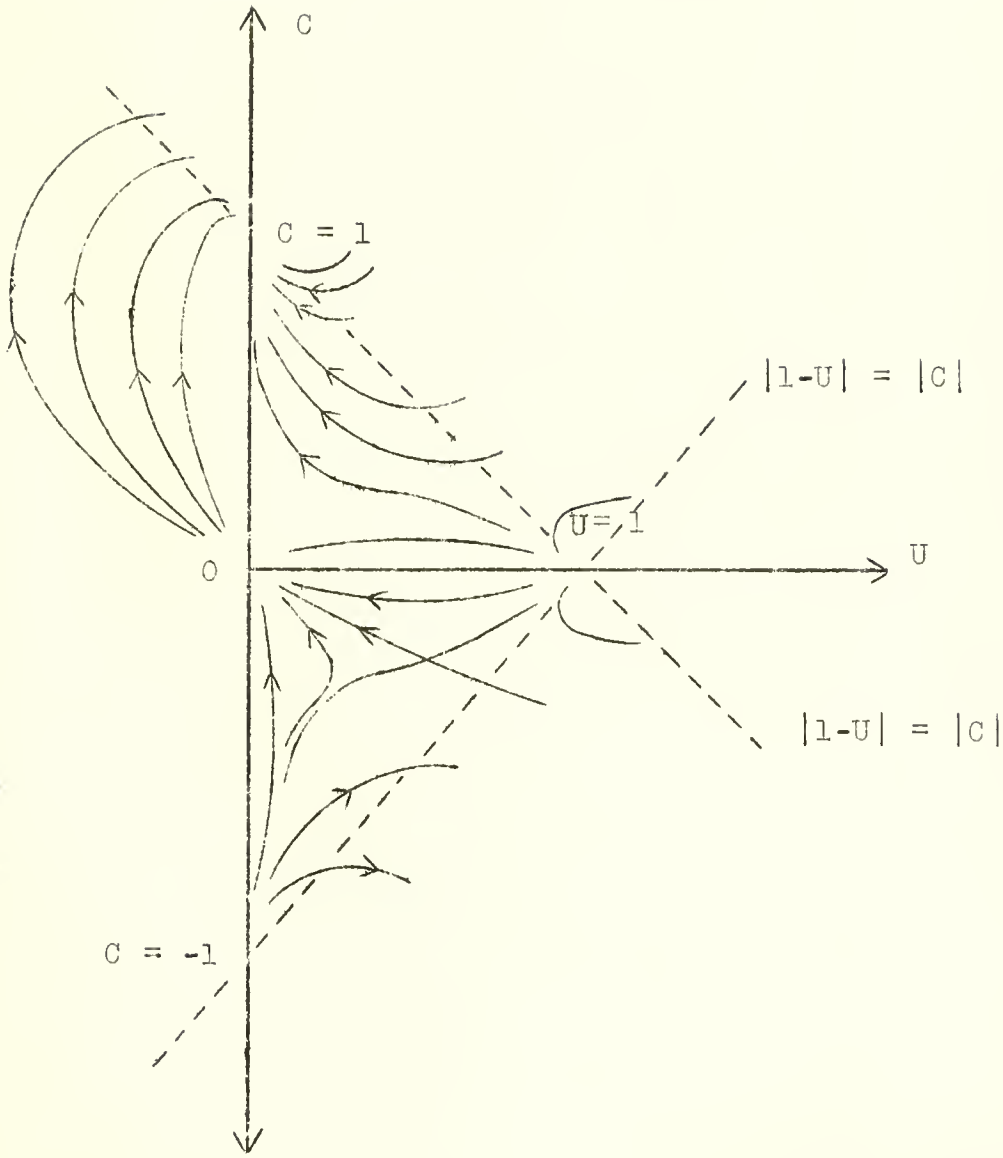

Figure 1

2. Shock Conditions and Corresponding Special Flow.

The conditions satisfied at a discontinuity in shallow water theory are, see [2],

$$Z_o(u_o - S) = Z_1(u_1 - S) \text{ (conservation of mass)} \tag{9}$$

$$Z_o(u_o - S)^2 + \tfrac{1}{2}gZ_o^2 = Z_1(u_1 - S)^2 + \tfrac{1}{2}gZ_1^2 \text{ (conservation of momentum).}$$

Here the subscript 0 will represent the state ahead of the shock, the subscript 1 the state behind the shock. The shock velocity is S.

A third condition is that there is to be a loss of energy across the shock. This is equivalent to requiring

$$|u_o - S| > C_o$$

or

$$|u_1 - S| < C_1 .$$

Setting $gZ = c^2$ again, we have

$$c_o^2(u_o - S) = c_1^2(u_1 - S) \tag{10}$$

$$c_o^2(u_o - S)^2 + \tfrac{1}{2}c_o^4 = c_1^2(u_1 - S)^2 + \tfrac{1}{2}c_1^4 .$$

These are equivalent to

$$c_1^2 = \frac{c_o^2}{2}\left(- 1 + \sqrt{1 + 8(u_o - S)^2 c_o^{-2}}\right)$$

$$u_1 - S = \frac{c_o^2}{c_1^2}(u_o - S) . \tag{11}$$

The suffixes may be interchanged to determine the state behind in terms of the state ahead.

In the case $u_o = 0$, (11) becomes

$$c_1^2 = \frac{c_o^2}{2}\left(-1 + \sqrt{1 + 8S^2c_o^{-2}}\right) \tag{12}$$

$$u_1 = S(1 - c_o^2c_1^{-2}) .$$

In the limiting case where $c_o \longrightarrow 0$ we have

$$c_1/S = 2^{\frac{1}{4}}\, c_o^{1/2}\, S^{-1/2}(1 + O(c_o)) \longrightarrow 0 \tag{13}$$

$$u_1/S = 1 - O(c_o) \longrightarrow 1 .$$

We note that we are led to the same result if we fix c_o and let $S \longrightarrow \infty$,

$$c_1/S = 2^{1/4}c_o^{1/2}(1 + O(S^{-1}) \tag{14}$$

$$u_1/S = 1 - O(S^{-1}) .$$

The special solution which corresponds to cavitation ahead, $c_o = 0$, can now be found as a progressive wave in the usual way. The shock is given by

$$\frac{dr}{dt} = S = \frac{r}{t} \tag{15}$$

or

$$r = k_1 t$$

or

$$\eta = k_1^{-1}$$

The shock conditions reduce by (13), (15) and (3) to

$$U = 1 \tag{16}$$

$$C = 0$$

We take any solution that starts from $U = 1$, $C = 0$ in the direction of increasing time and goes through $U = 0$, $C = 0$. In this section $C < 0$, since $t < 0$, $c > 0$. This solution is continued then through the origin. It is to be cut off by a reflected shock which will be determined. The state behind it, if it is to be of the form (3), must be the state of rest, $u = U = 0$. Otherwise we would be led to $d\eta < 0$ or cavitation.

The state of rest is given by

$$u = U = U_2 = 0 \tag{17}$$

$$c = c_2$$

$$\text{or} \quad C = C_2 = c_2 \eta$$

where c_2 is to be determined.

This state can be reached from states of the form (3) with a reflected shock given by $r = k_2 t$ provided U and C satisfy, by (11) and (3),

$$C^2(k_2)(U(k_2) - 1) = - c_2^2 k_2^{-2} \tag{18}$$

$$C^2(k_2) = \frac{c_2^2 k_2^{-2}}{2} \left(- 1 + \sqrt{1 + 8c_2^{-2}k_2^2}\right) .$$

These values of U and C lie on a curve passing through $(1, 0)$, $(0, \pm 1)$, and extending to $U = -\infty$ where it approaches the lines $C = \pm\sqrt{2}$ asymptotically. The portion of the curve where $|U - 1| > C$, that is $c_2 k_2^{-1} > 1$ corresponds to a state ahead of a shock.

The shock curve given by (18) intersects every solution curve passing through the origin into $U < 0$. To every intersection there corresponds a value of $c_2 k_2^{-1}$.

We are still left with a large family of solutions. In fact almost every solution that leaves $(1, 0)$ and enters $(0, 0-)$ is possible since they all end on the shock curve. However, the solution that leads to the lowest value of $c_2 k_2^{-1}$ -- that is, the weakest reflected shock, is not admissible since we cannot integrate $d\eta$. This solution leaves $(1, 0)$ and passes through the saddle point $(\frac{1}{2}, \frac{1}{2\sqrt{2}})$ where $\frac{d\eta}{dU} \longrightarrow \infty$. We may fix a solution by fixing the height at ∞.

In Figure 3, we indicate the behavior of one possible solution. This flow has been computed and is discussed in Section 4.

We must still determine η from (8) which we rewrite as

$$\frac{d\eta}{\eta ds} = \frac{dU}{Uds} + \frac{C^2}{U((1-u)^2 - 2C^2)}\frac{dU}{ds}$$

where

$$s = \int_{U=1, C=0}^{U, C} \sqrt{(dU)^2 + (dC)^2}\ .$$

Along the curves considered C^2/U is bounded as is $\dfrac{dU}{((1-U)^2 - 2C^2)ds}$. We may integrate to find η for $\eta < 0$

$$\log(-\eta) = \log U + g(s) + \log K_1$$

where $g(0) = 0$ and $\dfrac{dg}{ds} = \dfrac{C^2\, dU}{U((1-U)^2 - 2C^2)ds}$ and K_1 is to be fixed. Then

$$-\eta = K_1 U e^{g(s)} \tag{19}$$

At $U = 1$, $C = 0$ we have $\eta = k_1^{-1}$ by (15) and (16) and hence

$$-k_1^{-1} = K_1 .$$

As $U \longrightarrow 0$ we have

$$\eta^{-1} U \sim k_1 e^{-g(s_o)}$$

where $s_o = s(0, 0)$.

For $\eta > 0$ we have similarly

$$\log \eta = \log(-U) + g(s) - \log K_2 ,$$

or

$$\eta = -K_2 U e^{+g(s)} . \tag{20}$$

Near $U = 0$, $\eta = 0$ we have

$$-U\eta^{-1} \sim K_2^{-1} e^{-g(s)} .$$

If $u = \frac{r}{t} U = {}^{-1}U$ is to be continuous on $\eta = 0$ then we must have

$$K_2 \, e^{-g(s_o)} = K_1^{-1} e^{-g(s_o)}$$

or

$$K_2 = K_1 = - k_1^{-1} \; .$$

It is now possible to determine k_2. The line $\eta = k_2^{-1}$ corresponds to $U = U_1$, $C = C_1$ and let us say $s = s_1$. Then

$$k_2^{-1} = k_1^{-1} \, e^{g(s_1)} U_1 \; . \tag{21}$$

With this value of k_2, c_2 is determined from (18).

To summarize we may examine the solutions in the (r, t)-plane, see Figure 2:

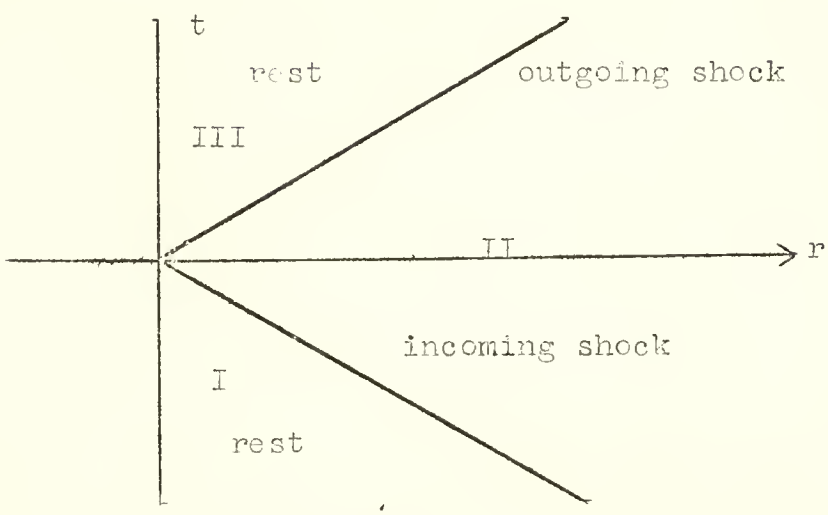

Figure 2.

In I we have cavitation $u = 0$, $c = 0$. In II the solution is given by (3), (19) and (20) where U and C are given by one of the solutions in the U, C-plane. The state III is again at rest with the water height given by $\frac{1}{g}c_2^2$.

The velocity k_2 of the reflected shock is related to k_1 the velocity of the incident shock by (21).

3. Finite Depth Problem.

We turn now to the problem in which the water has arbitrary finite depth ahead of the shock but the shock velocity becomes infinite at the origin. Certain conditions will be imposed on the shock path. Let the path of the incident shock be given by

$$r = R(t) \tag{22}$$

$$\text{or} \quad t = H(r) \quad \text{when} \quad H(r) < 0 .$$

We set

$$\begin{aligned} u &= \frac{H(r)}{t\frac{dH}{dr}}\, U\left(\frac{t}{H(r)}\right) \\ c &= \frac{H(r)}{t\,\frac{dH}{dr}}\, C\left(\frac{t}{H(r)}\right) \end{aligned} \tag{23}$$

in the region $k_2^{-1}H(r) > t > H(r)$ where k_2 is given by (21) with $k_1 = 1$. The reflected shock is given by

(24) $$k_2 t = H(r) .$$

The state behind the reflected shock is given by

(25) $$u = 0$$

$$c = c_2 \frac{dR}{dt} .$$

First let us show that (23) is an approximate solution of (2) near $r = 0$.

With $\eta = t/H(r)$

$$\frac{dH}{dr} = H .$$

(26) $$u_t + uu_r + 2cc_r = \frac{H}{\dot{H}t^2}[(-1-u)\eta U_\eta - 2C\eta C_\eta - U + (U^2 + 2C^2)$$

$$(1 - \frac{H\ddot{H}}{\dot{H}^2})]$$

$$\frac{2c_t}{c} + \frac{2uc_r}{c} + u_r + \frac{u}{r} = \frac{1}{t}[-\eta U_\eta + 2(1-U)C^{-1}\eta C_\eta + (3 + \frac{H}{\dot{H}r} - \frac{3\ddot{H}}{\dot{H}^2})$$

$$U - 2] .$$

Comparing (26) with (5) we see that the differential equations are approximately satisfied if

(27) $$|H\ddot{H}| \ll \dot{H}^2$$

$$|H - \dot{H}r| \ll \dot{H}r$$

These conditions are satisfied for example if we take as shock path near $t = 0$,

(28) $$r = R(t) = t \log(-t)$$

$$\text{or } r = H(r) \log(-H(r)) .$$

Then

$$1 = \dot{H}(1 + \log(-H))$$

$$\frac{H}{\dot{H}r} - 1 = \frac{H(1 - \dot{H}\log(-H))}{\dot{H}\,H\log(-H)} = \left(1 - \frac{\log(-H)}{1+\log(-H)}\right)\frac{1+\log(-H)}{\log(-H)}$$

$$= \frac{1}{\log(-H)} \longrightarrow 0 \quad \text{as} \quad r \longrightarrow 0$$

and

$$\ddot{H} = -\frac{\dot{H}^2}{H}\,\frac{1}{1+\log(-H)}$$

$$\text{or} \quad \frac{H\ddot{H}}{\dot{H}^2} = -\frac{1}{1 + \log(-H)} \longrightarrow 0 \quad \text{as} \quad r \longrightarrow 0.$$

At the shock the correct conditions would be given by (12) with $S = \dot{H}^{-1}$ but by (23), (22) and (16) we have

(29) $$u\dot{H} = 1$$

$$c\dot{H} \doteq 0 .$$

Comparing (29) and (14) we see we are neglecting terms

of order $\dot{H}^{1/2}$. Although the value of c itself at the shock is wrong, immediately behind the shock large terms of the order $\frac{H}{\dot{H}t}$ which behave like $\frac{r}{t}$ enter.

The reflected shock is given by $t = k_2^{-1}H(r)$. In the case of an incident shock of the form $R = t \log t$ this means that the reflected shock is given by

$$r = k_2 t \log k_2 t \quad k_2 t \log t .$$

At the reflected shock we have from (23),

$$u = \frac{k_2}{\dot{H}} U(k_2)$$

$$c = \frac{k_2}{\dot{H}} C(k_2)$$

and shock velocity $k_2\dot{H}^{-1}$.

The state immediately behind the reflected shock is then given correctly according to (11), in (25):

$$u = 0$$

$$c = c_2/\dot{H} = c_2 \frac{dR}{dt} k_2^{-1} \quad \text{for} \quad t = k_2^{-1}H(r).$$

In the whole region (III) the apparent discrepancy between (25) and (1) can be justified by considering the neglected terms as being of order $\dot{R}^{-1}$ where $\dot{R}(t) = dR/dt$. For example in the case where $R(t) = t \log t$, $\dot{R}^{-1} \longrightarrow 0$ as $t \longrightarrow 0$.

In fact if we consider solutions of the form

$$u = \frac{r\dot{R}(t)}{R(t)} U(R(t)/r)$$

$$c = \frac{r\dot{R}}{R(t)} C(R(t)/r) ,$$

we find again under the same restrictions on the shock path that U and C satisfy the ordinary differential equations (5). One solution corresponds to the state of rest, namely U = 0. Then

$$C = \text{constant.} \frac{R(t)}{r}$$

or $$u = 0$$

$$c = \text{constant.}\dot{R}$$

as in (25).

It can also be shown that the appropriate perturbation problem behind the reflected shock has a solution.

4. The Computed Flow.

A solution of (7) issuing from (0, 1) was computed as an example of a contracting shock. Let the velocity of the contracting shock be k_1. The solution chosen was the one for which the height at infinity, which is also the maximum height of the incident flow, is given by

$$Z_{max} = k_1^2 g^{-1} \lim_{\eta \to 0} \eta^{-2} C^2(\eta) = 1.59 g^{-1} k_1^2.$$

The the velocity at infinity as computed is

$$u_\infty = \lim_{\eta \to 0} \eta^{-1} U(\eta) = 2.12\ k_1 .$$

The velocity of the reflected shock is $1.65\ k_1$. The height behind the reflected shock is $7.48\ g^{-1}k_1^2$ or 4.7 times the maximum height of the incident flow.

In Figure 3, $gZ\eta^2 = C^2$ is plotted as a function of $u\eta = U$. The shock polar is the curve given by (12). In Figure 4, η is given as a function of U.

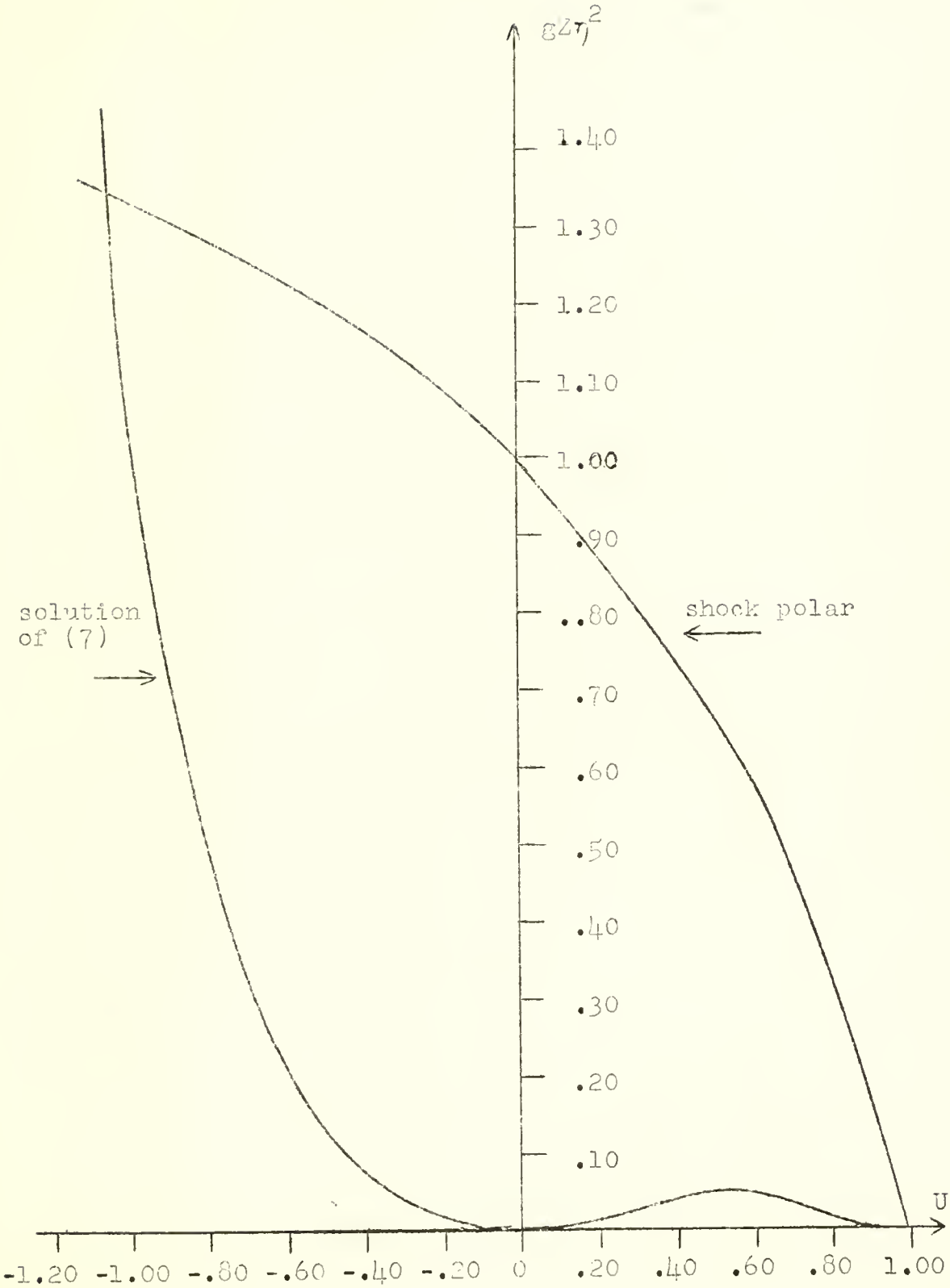
$8Z\eta^2$
1.40
1.30
1.20
1.10
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
solution of (7)
shock polar
U
-1.20 -1.00 -.80 -.60 -.40 -.20 0 .20 .40 .60 .80 1.00

Figure 3

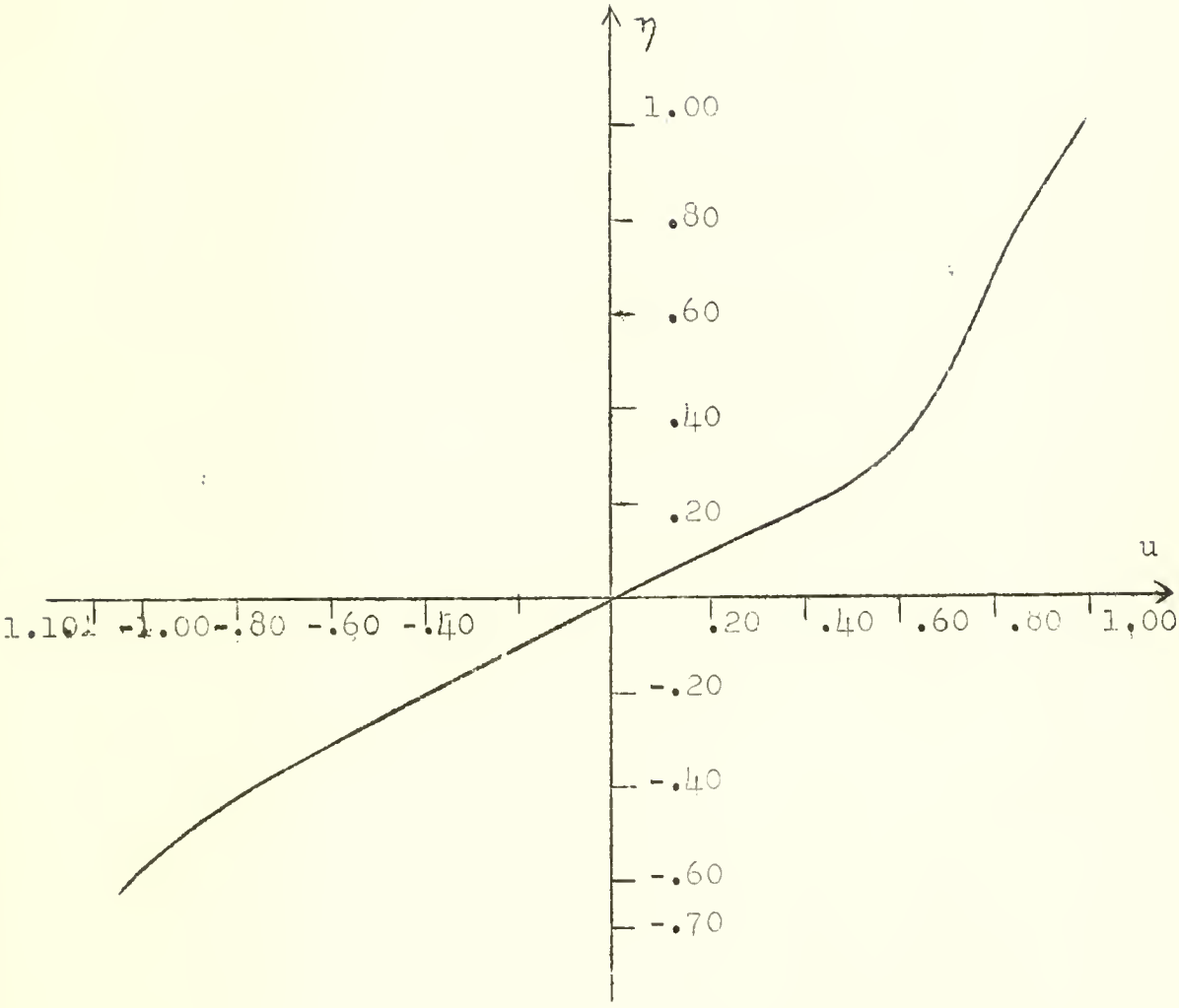

Figure 4

REFERENCES

[1] Guderley, G. Starke kugelige und zylindrische Verdichtungsstösse in der Nähe des Kugelmittel punktes bzw. der Zylinderachse. Luftfahrtforschung 19, No. 9.

[2] Stoker, J. J. The Formation of Breakers and Bores. Communications on Pure and Applied Mathematics, Vol. 1, No. 1, 1-87 (1948).

[3] Courant, R. and Friedrichs, K. O. Supersonic Flow and Shock Waves. Interscience Publishers (1948).

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